ClassPad 101

for ClassPad Version 3.00+

Lesson 22

Introduction to Calculus on the ClassPad

Welcome

In this lesson we will explore features of the ClassPad that allow us to study some areas of Calculus. This lesson is written with the assumption that the student has had at least Pre-Calculus. However, all the steps are given and everyone is encouraged to try it!

Lesson Goals

- To understand how to find limits from the left and right
- To be able to visualize limits
- To understand the relationship between even and odd functions and definite integrals
- To be able to visualize the derivative of a circle

In Lesson 22, you will learn how to:

- Study the result of a limit algebraically and graphically
- Apply properties of even and odd functions to definite integrals
- View the slope of the tangent line
- Create an animation of a tangent line on a curve
- Create an animation table
- Plot data created from an animation

Upon completion of this lesson, you will be able to answer the following questions:

1. How do we find the limit of a function from the left on the ClassPad?
2. How do we find the limit of a function from the right on the ClassPad?
3. True or False: If we replace x with –x and get the same function, then the graph of the function is symmetric about the y-axis.
4. What is the integral of f(x) from -3 to 3 if f(x) is odd and well defined between -3 and 3?
5. How can we view the equation of a tangent line in Geometry?
6. How can we estimate the graph of a derivative in Geometry?

Time required

About 75 minutes.

Getting Started

We will begin with limits. It is simple to find a limit with the ClassPad. However, sometimes the answer may not make sense and the answer should always make sense. Question everything until you understand! Limits are wonderful to work with on the ClassPad because we can drag the function to a graph window to better understand the limit. Always explore to understand.

PART I

1. Setting up a Limit

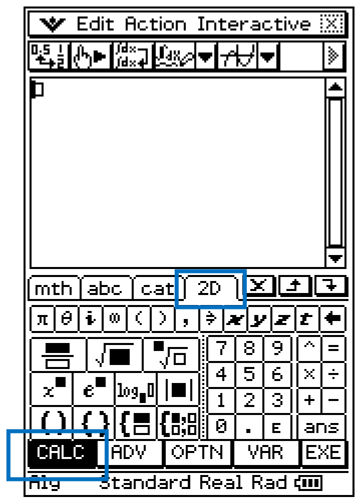
a. Open Main and clear the window (if needed)

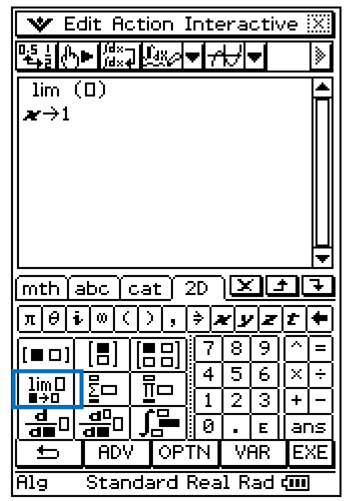
b. Open the Keyboard to the **2D** tab

c. Click **CALC**

d. Click the **2D limit** form

e. Input ***x*** and **1** for "the limit as x approaches 1"





2. Experimenting with a Limit

a. Click the ⮌ key to find the fraction key

b. Using the fraction key, input $\dfrac{1}{x-1}$

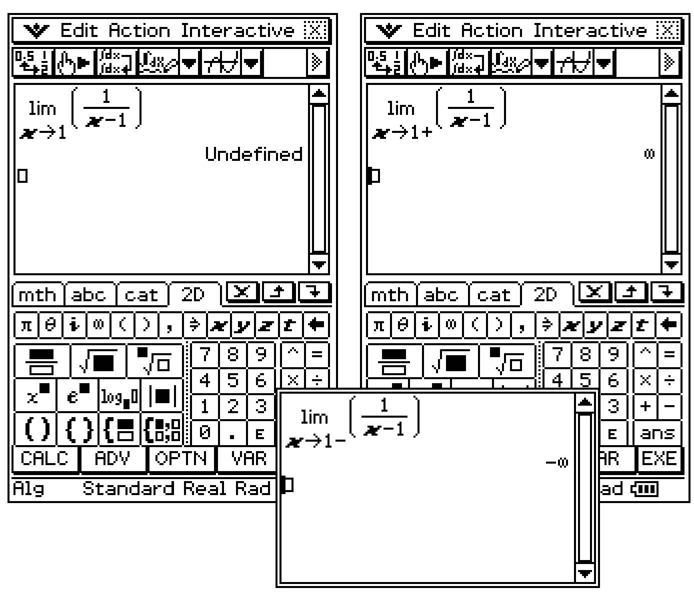
c. Press **EXE** (hmm…)

d. Click just past the 1 and input a + sign

e. Press **EXE**

f. Select the + sign and change it to a – sign

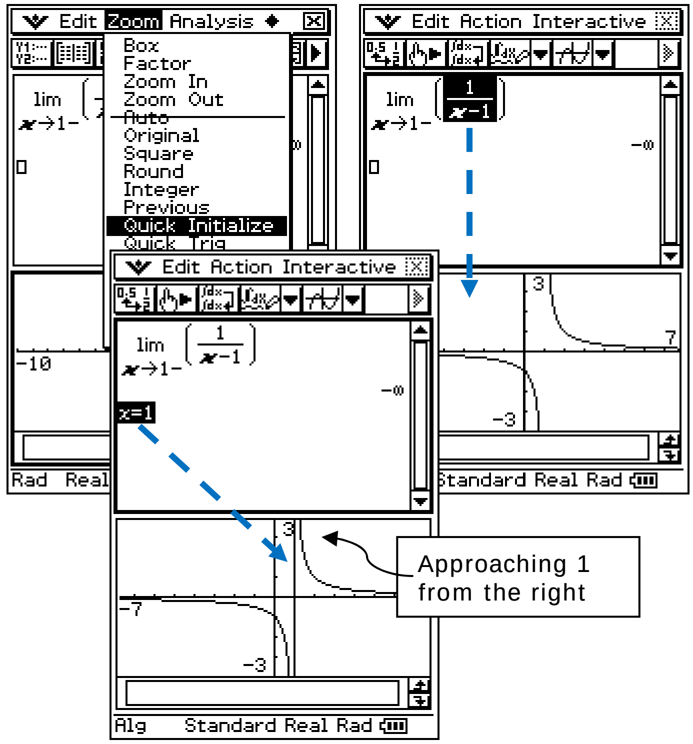
g. Press **EXE**



3. Viewing Limits Graphically

a. Click on the toolbar to input a graph window
b. Open the **Zoom** menu and select **Quick Initialize**
c. Select just $\frac{1}{x-1}$, let go, then press and drag to the graph window
d. Input $\boldsymbol{x=1}$ *BUT do not* press EXE

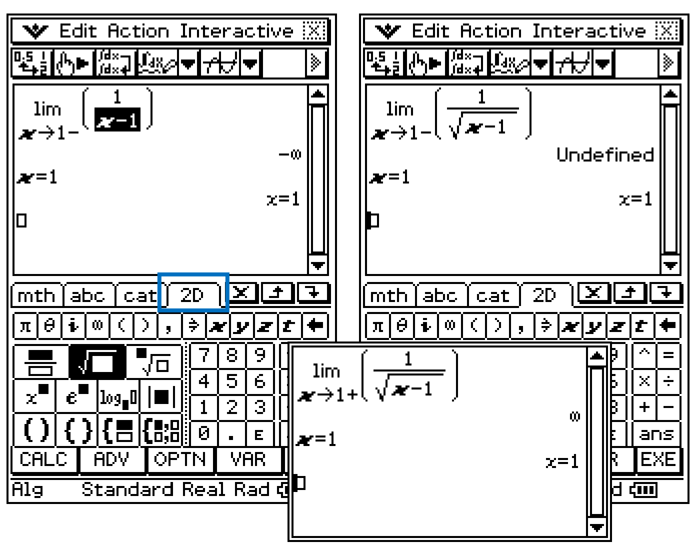
   Note: If you do press EXE, the graph window will clear before graphing the next dropped function. This is ok. You can graph the original again!
e. Select $\boldsymbol{x=1}$, let go, then drag to the graph window
f. Visualizing limits is fun!



Since the limit from the left ($\lim_{x\to 1^-}\left(\frac{1}{x-1}\right)$) does not equal the limit from the right ($\lim_{x\to 1^+}\left(\frac{1}{x-1}\right)$), the limit ($\lim_{x\to 1}\left(\frac{1}{x-1}\right)$) does not exist. The graph displays this nicely!

4. Viewing Other Limits

a. Click in the **Main** window and open the **keyboard**
b. Select x-1 and click to change $\frac{1}{x-1}$ to $\frac{1}{\sqrt{x-1}}$
c. **Press** the button to execute one line (Why Undefined?)
d. Click just past the 1 and input a + sign
e. Click or press EXE
f. Examine the limit results by graphing $\frac{1}{\sqrt{x-1}}$



PART I
Practice Exercises

Before beginning the practice exercises, open a word document, type in the following information and then *save it as Lesson22 in your CASIO folder within My Documents*:

- Date: (enter today's date)
- To: (put your instructor's name here)
- From: (put your name here)
- Re: Lesson 22

1. Please begin by opening the **Main** application. Clear the window.
2. Using the 2D Limit notation, evaluate the limit of $\sqrt{x^2-4}$ as **x** approaches 2 from the right.
3. With your limit showing, get a **screen capture**. Paste it into your Lesson22 document (under a title of PART I).
4. Change your input to find the limit as **x** approaches 2 from the left.
5. With your limit showing, get a **screen capture.** Add two blank spaces following the first screen capture and then paste this one.
6. Insert a graph window. Using drag and drop, graph $\sqrt{x^2-4}$ and also x=2.
7. With the graph window showing, get a **screen capture.** Add two blank spaces following the last screen capture and then paste this one.

PART II

In this part, we will begin by reviewing even and odd functions and then consider examples involving integration.

1. Defining Functions

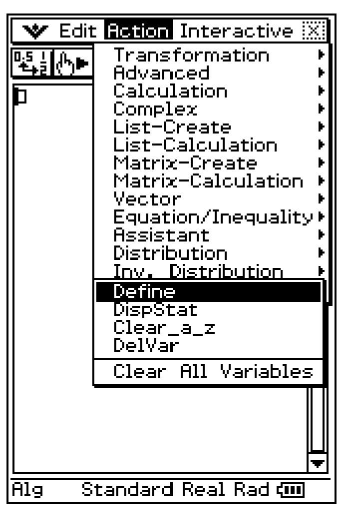
a. Open **Main** and clear the window (if needed)

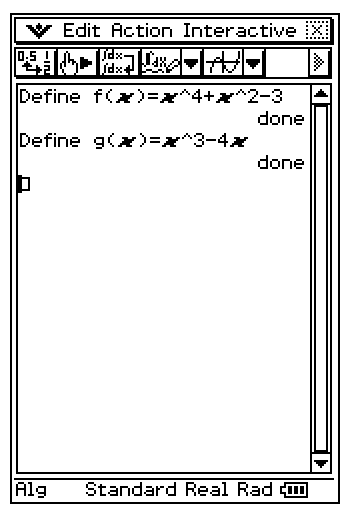
b. Select **Action/Command/Define**

c. Input **f(x)=x^4+x^2-3** and press **EXE**

d. Select Define and drag it to the next line or use the Action menu

e. Input **g(x)=x^3-4x** and press **EXE**

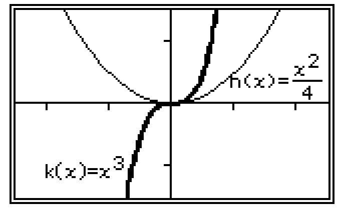




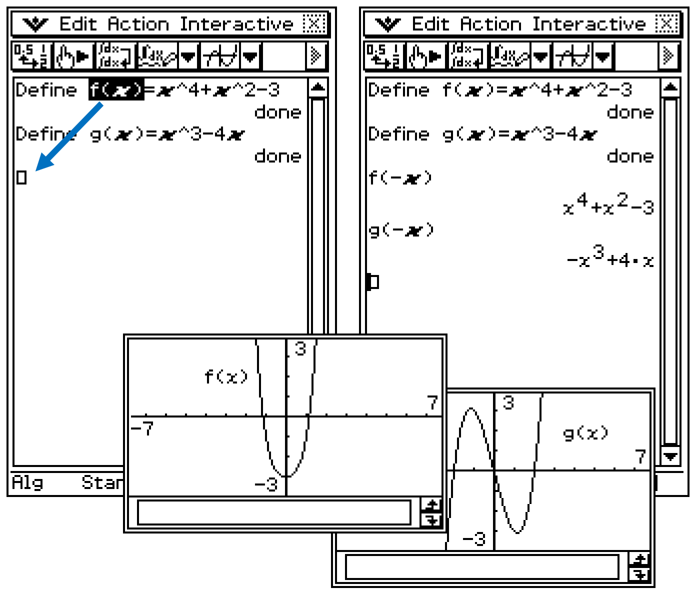
2. Even and Odd Functions

Recall:
If $f(-x)=f(x)$, then f is said to be an **even function**.
Even functions are symmetric about the y-axis.

If $f(-x)=-f(x)$, then f is said to be an odd function.
Odd functions are symmetric about the origin.

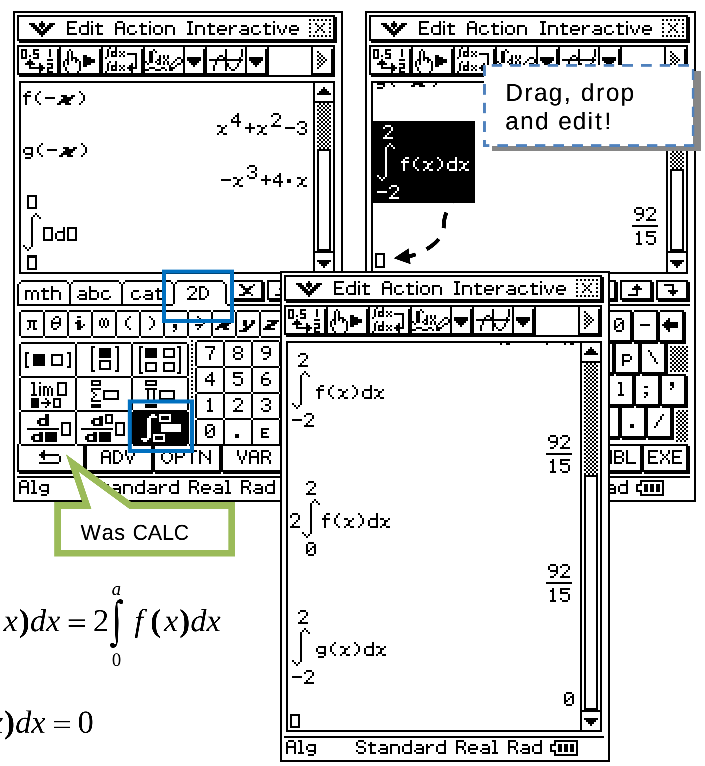


a. Select $f(x)$ and drag down to the **next open line**
b. Change $f(x)$ to $f(-x)$ and press **EXE**
c. Repeat steps a and b for $g(x)$
   - Does $f(x) = f(-x)$?
   - Does $-f(x) = f(-x)$?
d. Click on the toolbar to input a graph window
e. Select $f(x)$ and drag it to the graph window
f. Repeat with $g(x)$



3. Integrating Even and Odd Functions

a. Click in the **Main** window and open the **keyboard**
b. Open the **2D page** and click **CALC** to find
c. Integrate $f(x)$ from -2 to 2
d. Next, integrate $f(x)$ from 0 to 2 **AND** multiply the integral by a factor of 2
   - *Why* are these integrals equal? Consider the graph!
e. Integrate $g(x)$ from -2 to 2
   - *Does* this result make sense?



Recall:

If f is an *even* function, then $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

If f is an *odd* function, then $\int_{-a}^{a} f(x)dx = 0$

4. Investigating Non-Even and Non-Odd Functions

a. **Change** the Define ***f(x)*** to: ***x***^4**–*x*–**3

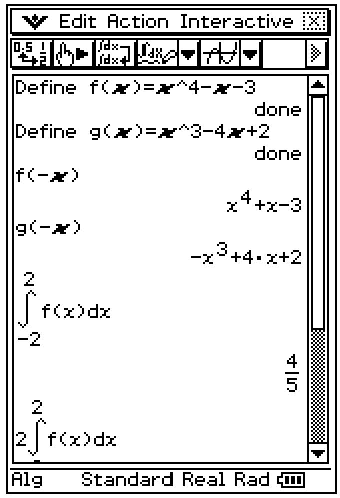
b. **Press** the button

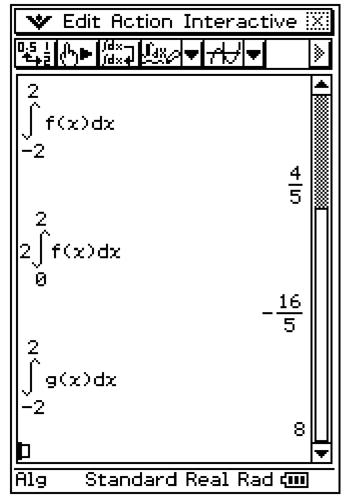
c. **Change** the Define ***g(x)*** to: ***x***^3***–4x+2***

d. Press **EXE** to re-execute everything

Consider:

- Is ***f*** still an even function?
- Is ***g*** still an odd function?





5. Continuing the Investigation of Non-Even and Non-Odd Functions

a. Click on the toolbar to input a graph window

b. Open the **Zoom** menu and select **Quick Initialize**

c. Select just f(x) and drag to the graph window

d. Select **Analysis/G-solve/∫dx**

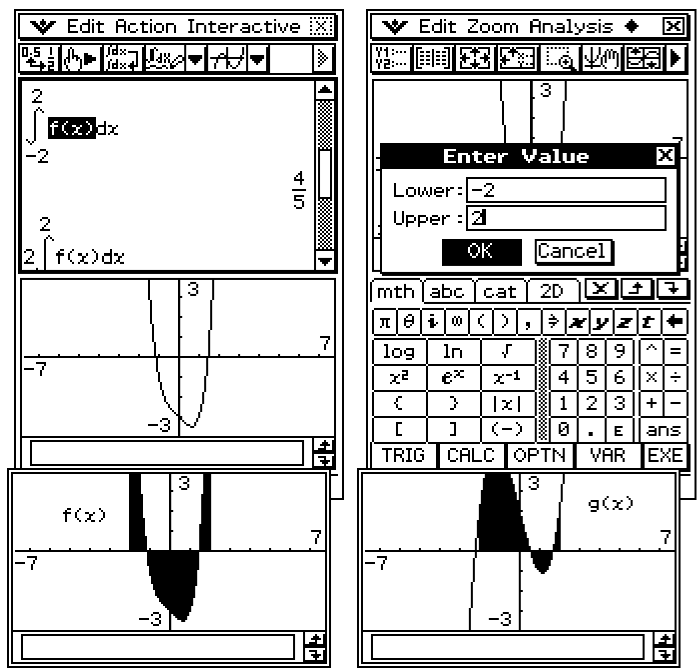
e. **Type** -2 (<u>*a dialog will open automatically*</u>)

f. Input an upper limit of 2 and click OK

- What do you think of the area?

g. Clear the graph window (Edit/Clear All while the graph window is active!)

h. Repeat for g(x)



PART II
Practice Exercises

1. Please begin by opening the **Main** application. Clear the window.
2. Using the Define command, define f(x)=x^4+2x^3-3x+2 and press **EXE**.
3. On the next input line, evaluate f(-x).
4. On the next line, find the integral of f(x) from -1 to 1 and press **EXE**.
5. On the next line, find the integral of f(x) from 0 to 1 and multiply the integral by a factor of 2. Press **EXE**.
6. With the Main window showing, get a **screen capture**. Paste it into your Lesson22 document (under a title of PART II).
7. Modify f(x) to be an even function. You can modify it however you like (add, change or delete terms and/or coefficients) to make it even. [Hint: Consider the exponents.]
8. When you have f(x) modified to be even, press EXE from the top line to re-execute everything.
9. With the results of your even function showing, get a **screen capture**. Add two blank spaces following the first screen capture and then paste this one.
10. Modify f(x) to be an odd function. Again, consider the exponents.
11. When you have f(x) modified to be odd, press EXE from the top line to re-execute everything.
12. Insert a Graph window.
13. Graph f(x). You can use the + or – key to zoom if needed.
14. Using the Analysis/G-Solve menu, find the area from -1 to 1.
15. With Main and the shaded graph area showing, get a **screen capture**. Add two blank spaces following the second screen capture and then paste this one.

PART III

In this part, we will look at the derivative of a circle using Main and the animation feature in the Geometry application.

1. Drawing a Circle

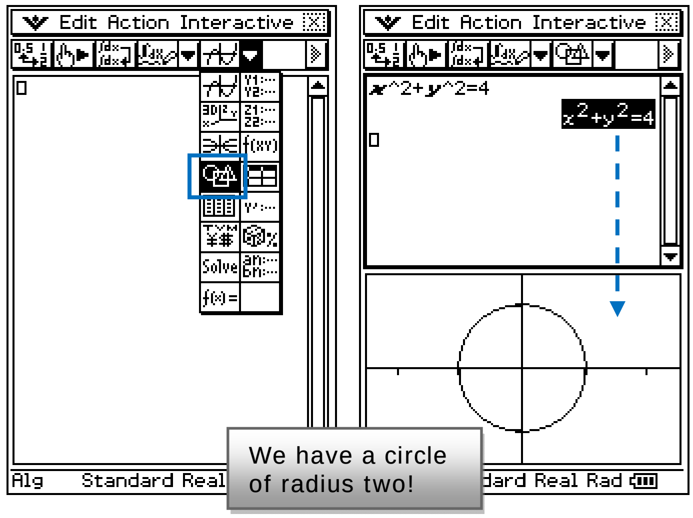
a. Open Main and clear the window (if needed)

b. Insert a Geometry window and select **File / New**

c. Click to turn the axis on (if needed)

d. In Main, input: **X^2+y^2=4** and press **EXE**

e. Select the output, let go and then press and drag to Geometry



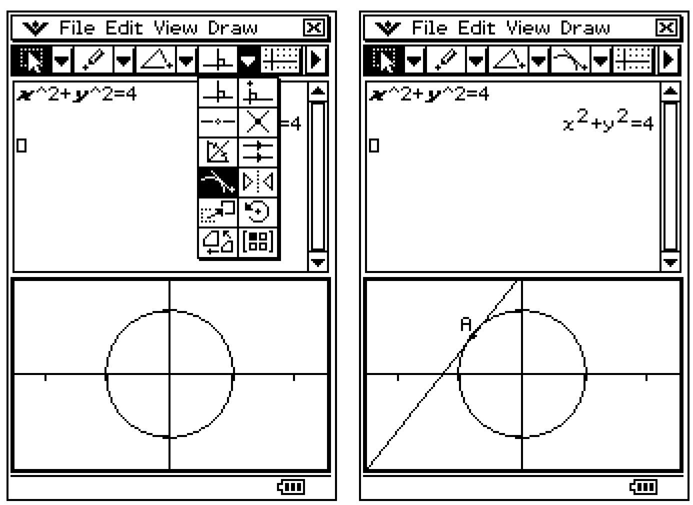
2. Drawing a Line Tangent to the Circle

a. Click in **Geometry** to give it focus (if needed)

b. **Click** the fourth and select (tangent to curve symbol)

c. **Click on the circle** and let go!

d. **Resize** the **Geometry** window (Ctrl+r or Resize)



3. Viewing the Equation of the Tangent Line

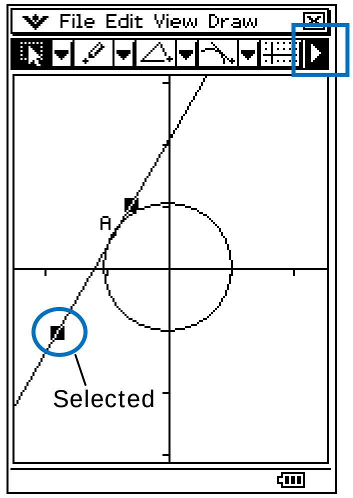
a. Click on the line only to select it

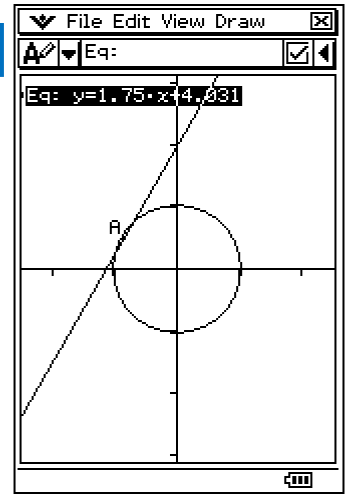
b. **Click** [▶] to advance the toolbar

c. Click on [x+y=] to place the equation in the Geometry window

d. Position the equation to better view the slope

e. Click in any white space to deselect everything





f. **Select** the circle only

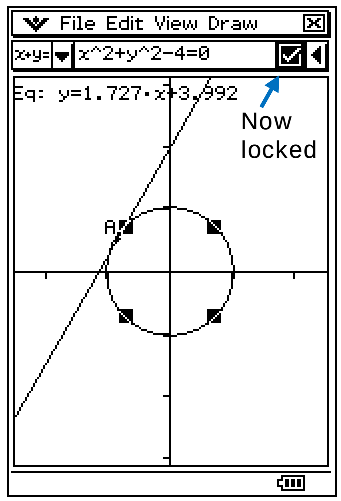
g. With [x+y=] showing, click [☑] to **lock** the equation

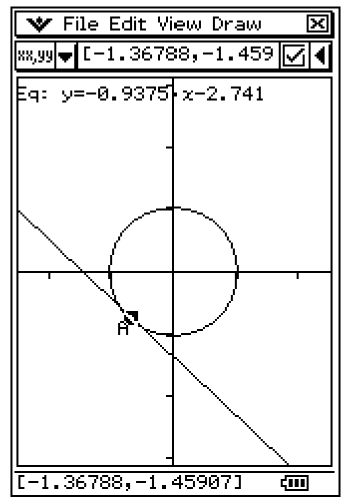
h. Click in any white space to deselect everything

i. Click on pt A to select it and let go

j. Press on the selection and drag pt A to another location on the circle

Notice the displayed equation updates automatically!





4. Adding and Running an Animation

a. Click in any white space to deselect everything

b. Select **pt A** and then **the circle**

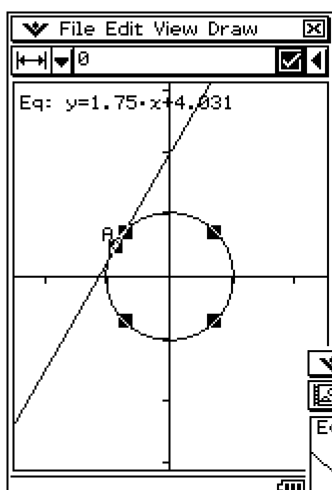
c. Select **Edit / Animate / Add Animation**

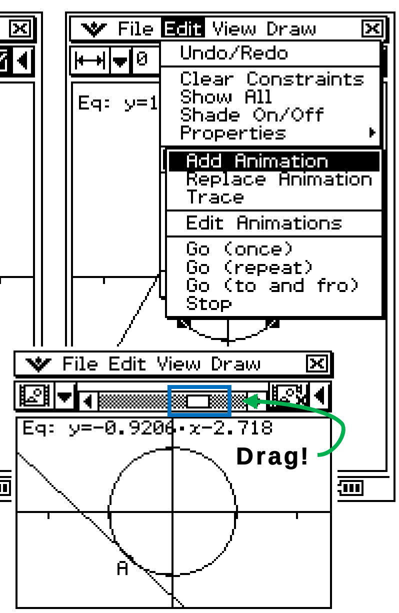
d. Select **Edit / Animate / Go (once)**

e. Another way: Select **View / Animation UI**

f. Press on the thumb and drag to move the tangent line

g. Select **View / Animation UI** to remove it from the toolbar





5. Viewing the Derivative of the Circle

When we add an animation, data related to the animation is available for us to use. We will view the data for the coordinates of pt A as it moves about the circle and for the slope of the tangent line. We will then plot the x-coordinates and corresponding slopes with the graph of the circle!

a. Click to advance the toolbar

b. Click in any white space to deselect everything

c. Select **pt A** and make sure is showing

d. **Click** to add the coordinates to a table

e. Click in any white space to deselect everything

f. Select the tangent line

g. If is not showing, **click** and select (slope symbol)

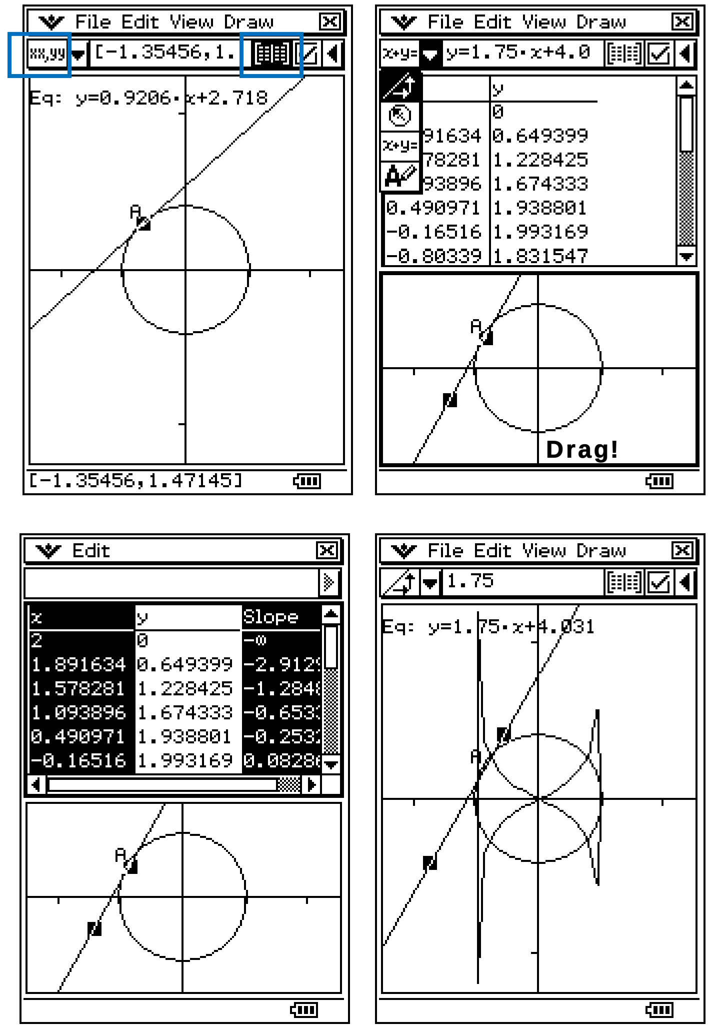
h. **Click** to add the slopes to a table

i. Click on **column x** and then **column Slope**

j. **Note:** Click on a column again to deselect it

k. Press on ***either*** selected column and drag to the Geometry window

l. Resize the window

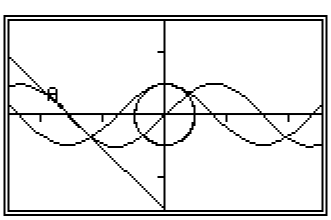


Note: What we see are the actual data points plotted giving us an estimation of the derivative only. The end points connecting is a limitation of plotting points. Does this estimate make sense? Think about the slope of the tangent line as it moves about the circle. What happens at (2,0), (0,2), (-2,0) and (0,-2)? What happens within each quadrant? Try sketching a better graph of the derivative of a circle!

PART III
Practice Exercises

1. Open Main and clear the window.
2. Input: y=sin(x) and press EXE.
3. Insert a Geometry window.
4. Select **File / New** and turn the axis on.
5. Select y=sin(x) and drag it to the Geometry window. This curve should look nice! If not, make sure you are in Rad mode.
6. Draw a tangent line to the sine curve (anywhere).
7. Select the point of tangency and the sine curve.
8. Select Edit/Animate/Add Animation and then Edit/Animate/Go (once).
9. Advance the toolbar.
10. Select the point of tangency and make sure the coordinate button is showing.
11. Click the table icon. A table should appear.
12. Deselect everything in Geometry and then select the tangent line.
13. Make sure the slope button is showing and then click the table button.
14. With the Geometry window and table showing, get a **screen capture**. Paste it into your Lesson22 document (under a title of PART III).
15. Click on the **x** column and **slope** column to select them.
16. Press on either column and drag to the Geometry window.
17. With the Geometry window and table showing, get a **screen capture**. Add two blank spaces following the first screen capture and then paste this one.
18. Click on the **x** column to deselect it.
19. Click on the **y** column to select it. You should now have the y column and Slope column selected.
20. Press on either column and drag to the Geometry window.
21. With the Geometry window and table showing, get a **screen capture**. Add two blank spaces following the second screen capture and then paste this one.

   Your Geometry window should look similar to:



PART IV

Reflection Exercises

You have just completed the twenty-second lesson in ClassPad 101. Please take a few moments to copy and paste the following three questions at the end of your Lesson22 document and answer them.

1. Approximately how long did it take you to complete this lesson?
2. Which activity did you enjoy the most?
3. Did you find any part of this activity difficult to follow? If so, which part? Also, how did you overcome the difficulty?

Assessment 22: Introduction to Calculus on the ClassPad

- **Checkpoint**: Your word processed document, titled "Lesson22", should contain the following activities:
  1. Three screen captures from PART I.
  2. Three screen captures from PART II.
  3. Three screen captures from PART III.
  4. Three reflection questions with answers from PART IV.

- **Submit** your **Lesson22 document** to your instructor for grading. Once your lesson is submitted, your lesson for ClassPad 101 "Introduction to Calculus on the ClassPad" is complete.