ClassPad 101

for ClassPad Version 3.00+

Lesson 18
Introduction to Sequence

Welcome
What do you think of when you hear the word “sequence”? Do you think of a set of numbers listed in a special order? If we can find a pattern for the set of numbers or sequence, we can write an algebraic form called a *closed form* for the sequence. If we have a closed form for a sequence, we can analyze it using the Sequence application.

Lesson Goals
- To understand the closed form of a sequence
- To be able to identify a given sequence as arithmetic or geometric
- To understand the relationship between a sequence and a series
- To understand the summation symbol

In Lesson 18, you will learn how to:
- Input sequences in explicit and recursive formats
- View data tables for sequences
- Graph sequences
- Identify whether a sequence is arithmetic or geometric
- Use summation notation

Upon completion of this lesson, you will be able to answer the following questions:
1. If the difference between consecutive terms is the same, what type of sequence is it?
2. What type of sequence requires us to have an initial term given?
3. How do we link a table of values to the graph window?
4. How many terms in a sequence are summed if the starting value is n=12 and the ended value is n=21?

Time required
About 60 minutes.

Getting Started

We will begin with a very quick introduction to sequences. A general understanding of the terminology used in writing sequences will help you in using the Sequence application. As you learn about sequences in school, you will find the Sequence application particularly useful.

Sequences are defined with a useful notation that contains subscripts. We input a number from a set {1, 2, 3 …} to get a term in the sequence.

Notation (Mathematical language is interesting!)

a_n is read a sub n
a_k is read a sub k

The open form of an infinite sequence can be written as:

$\{ a_1, a_2, a_3, a_4, \ldots, a_n, a_{n+1}, \ldots \}$

The closed form of a sequence (if we can find one) is an equation that can be used to generate each term in the sequence.

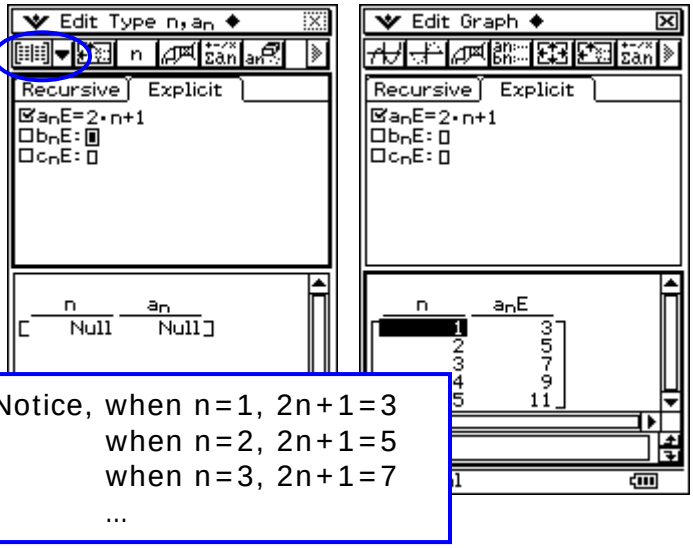
For example,
If the closed form of a sequence is given by:
$\mathbf{a_n = 5 - n}$
then the sequence can be found by replacing n with 1, then 2, then 3, …
For n=1, we have $a_1 = 5-1 = 4$
For n=2, we have $a_2 = 5-2 = 3$
For n=3, we have $a_3 = 5-3 = 2$
And so on!

PART I

In this part, we will learn how to use the Sequence application to view a Sequence in tabular and graphical forms.

1. Entering a Sequence

a. Open Menu and then Sequence
b. Click on the **Explicit** tab and click following $\mathbf{a_nE}$
c. Type in **2n+1** and press **EXE**
d. Click the table icon ( ) [Hint: If it is not showing, click the ▼ to find it.]



Notice, when n=1, 2n+1=3
when n=2, 2n+1=5
when n=3, 2n+1=7
…

2. Working with a Sequence Table

a. To change the size of the table, click 

b. Change **End** to **10** and click **OK**

c. **Click** back in the Sequence Editor window

d. **Click**  and select 

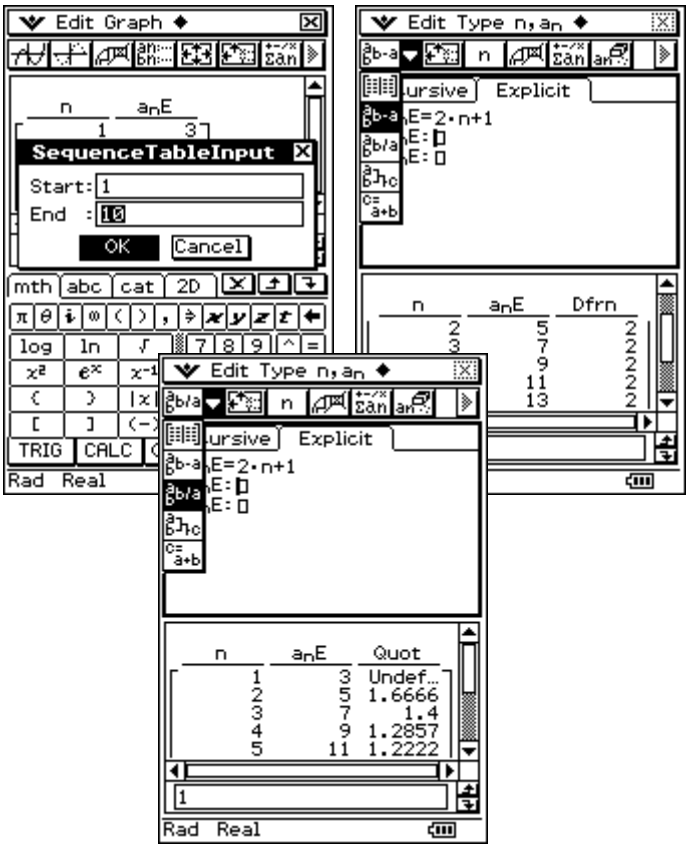
e. This button adds the **difference** of two consecutive values to the table.

f. Note: Since the differences are constant, our sequence is an **Arithmetic sequence**

g. **Click**  and select 

h. This button adds the **quotient** of two consecutive values to the table.

i. Note: If the quotients were constant, our sequence would be a **Geometric sequence**



3. Working with a Sequence Graph

a. **Click**  again to display just the data in the table

b. **Click**  to plot the data in the table

c. **Click** back in the **Table** window

d. **Click**  to connect the dots

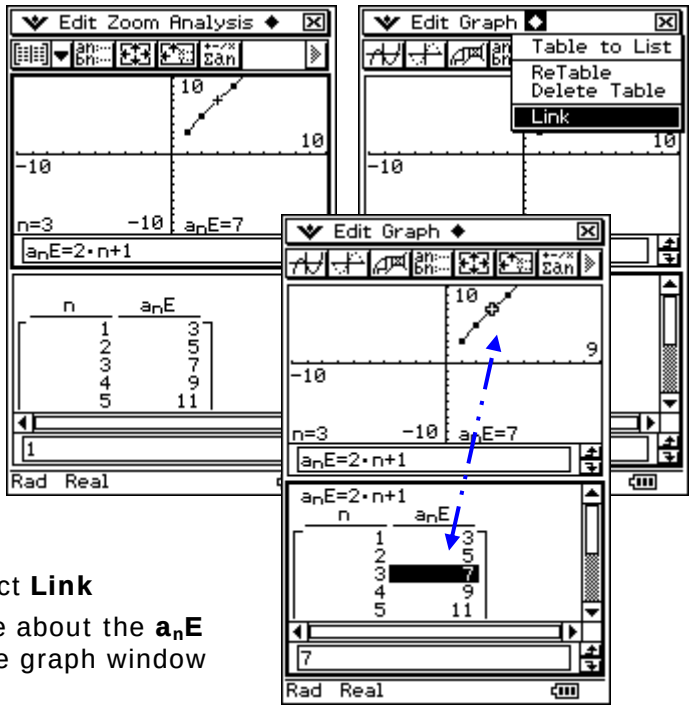
e. **Open Analysis** and select **Trace**

f. Use the arrow keys to move from point to point

g. Another way to trace: **Click** back in the **Table** window

h. **Open** the ◆ menu and select **Link**

i. Use the arrow keys to move about the **a_nE table values** and watch the graph window



PART I
Practice Exercises

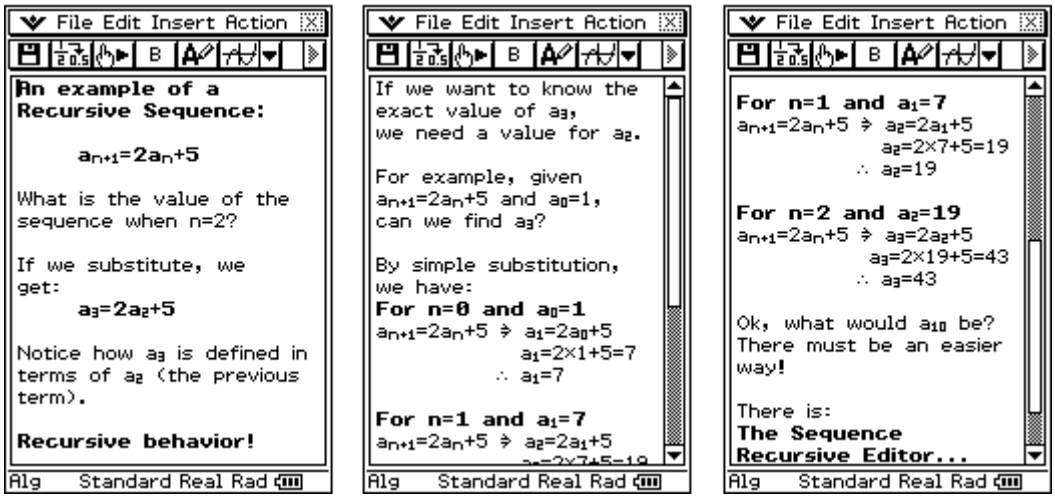
Before beginning the practice exercises, open a word document, type in the following information and then *save it as Lesson18 in your CASIO folder within My Documents*:

- Date: (enter today's date)
- To: (put your instructor's name here)
- From: (put your name here)
- Re: Lesson 18

1. Please open the Sequence application and clear the Explicit page.
2. Generate a table of values for the sequence $\mathbf{a_n = 2(3/4)\text{^}n}$.
3. Increase the table size so that it shows values from **1** to **20.**
4. With the sequence and table showing, get a **screen capture**. Paste it into your Lesson18 document (under a title of PART I).
5. **Plot** the data in your table using the dot graph style. Open the **Zoom menu** and select **Auto**.
6. With the graph and table windows showing, get a **screen capture.** Add two blank spaces following the first screen capture and then paste this one.
7. Add a quotient column to your table (the column heading will be Quot).
8. Resize the Table window so that it is maximized. Notice the Quot column is constant which means our sequence is a Geometric sequence. Also notice that as **n** increases, $\mathbf{a_n}$E seems to be approaching a constant value. You will learn more about this in Calculus.
9. With the Table window maximized, get a **screen capture.** Add two blank spaces following the last screen capture and then paste this one.

PART II

In this part, we will work with sequences written in a recursive form. A sequence is written in a "recursive form" if its definition uses the previous term.



In short, with recursive sequences, we need to be given at least one value in the sequence to find the values of other terms in the sequence.

1. Entering a Recursive Sequence

a. Open Menu and then Sequence

b. Click on the **Recursive tab** and click following $\mathbf{a_{n+1}}$

c. We will input $\mathbf{2a_n+5}$: Type **2,** open the **n, a_n** menu and select $\mathbf{a_n}$, type **+5**

d. Press **EXE**

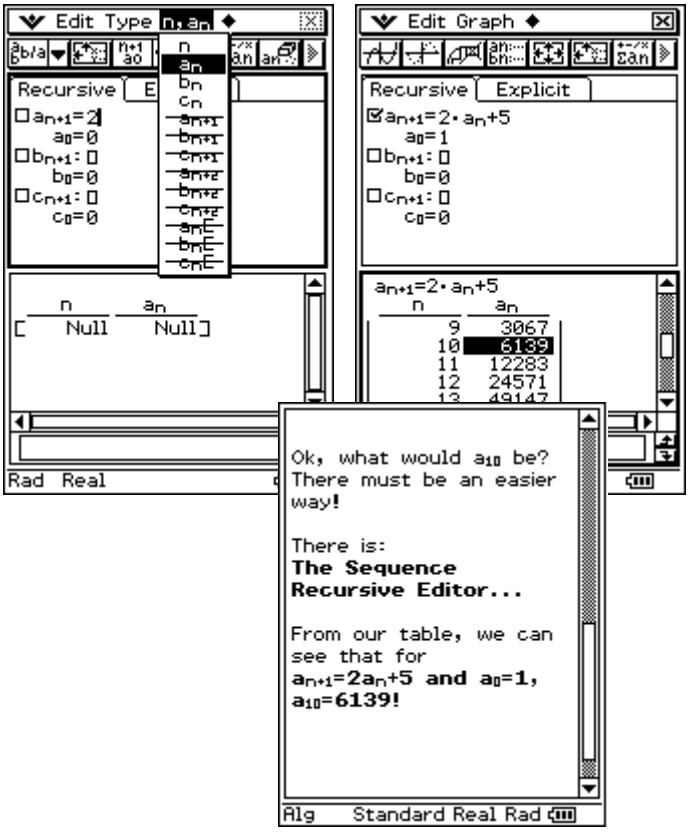
e. We NEED to define $\mathbf{a_0}$

f. Set $\mathbf{a_0=1}$ and press **EXE**

g. **Click** ▼ and select the table icon (▦)

h. Scroll the Table window

i. Practice graphing and tracing the data



2. Entering Another Recursive Sequence

a. Click a_n:: b_n::

b. Click following $\mathbf{b_{n+1}}$

c. Input $\mathbf{b_n}$ **+n** and press **EXE**

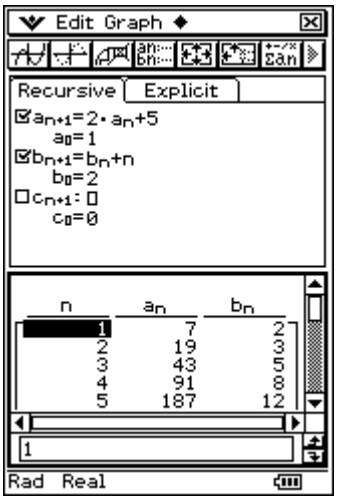
d. Set $\mathbf{b_0}$=2 and press **EXE**

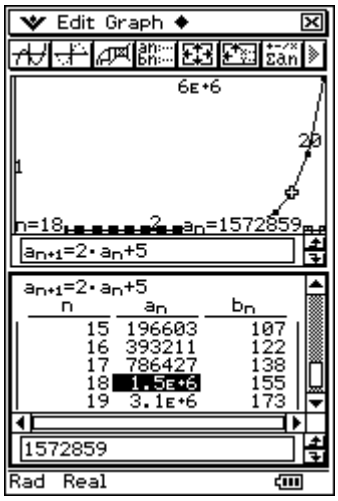
e. Click the table icon ( )

f. **Graph** and select **Zoom / Auto**

g. **Click** back in the **Table window**

h. **Open** the ◆ menu and select **Link**





i. Use the arrow keys to move about the $\mathbf{a_n}$ and $\mathbf{b_n}$ table values (notice the cursor jumps from one graph to the other)

3. Finding the Explicit Form of a Recursive Sequence

The Sequence application has a special math window that looks like the Main application, but with different menus. The menus contain items commonly used with Sequences.

Sometimes it is nice to rewrite recursive sequences as explicit sequences. Why? Well, explicit sequences do not depend on previous terms. We will use the rSolve( ) command. This command tries to find an equivalent sequence for our recursive sequences in an explicit form.

a. Click a_n:: b_n::

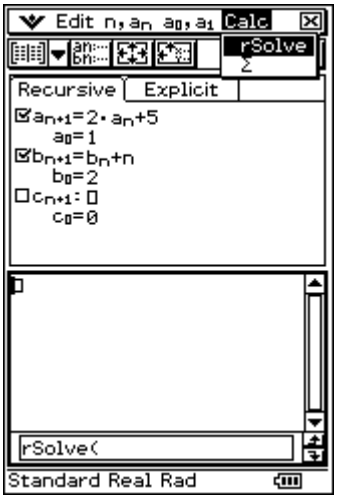
b. **Click** the Σan button

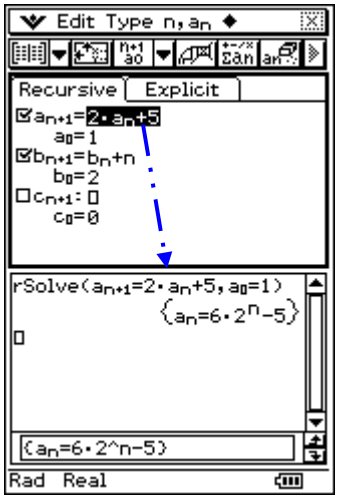
c. Open the **Calc menu** and select **rSolve(**

d. Look in the **n,$\mathbf{a_n}$** and **$\mathbf{a_0}$,$\mathbf{a_1}$** menus (just look)

e. Using menu items and drag&drop, type in the recursive form for $\mathbf{a_{n+1}}$ (as shown)

f. Press **EXE**





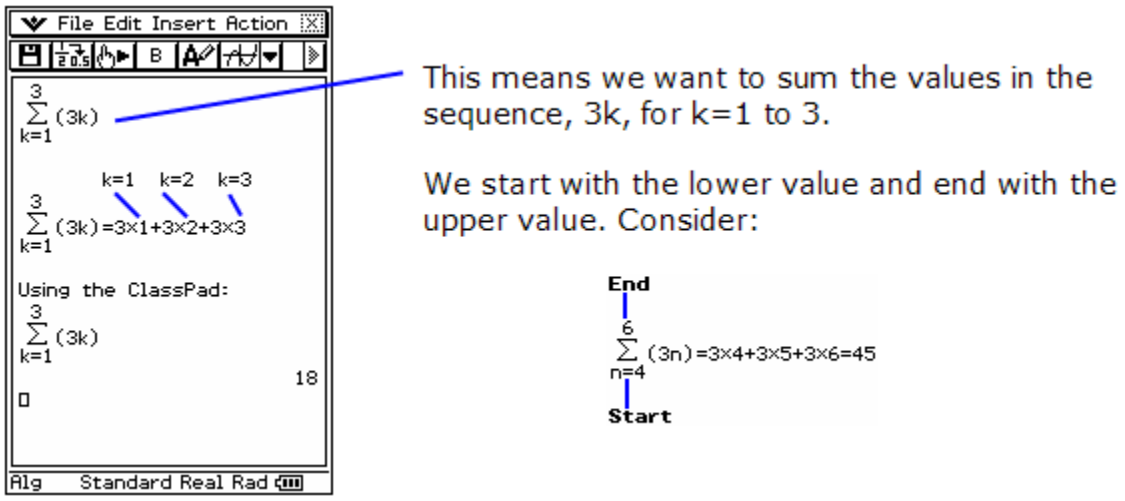
PART II
Practice Exercises

1. Please open the Sequence application and clear the Recursive page.
2. Generate a table of values for the sequence $\mathbf{a_{n+1} = n - 2a_n}$, $\mathbf{a_0 = 15}$.
3. With the sequence and table showing, get a **screen capture**. Paste it into your Lesson 18 document (under a title of PART II).
4. Using the **rSolve( )** command; find the explicit form for our recursive sequence.
5. With your rSolve result showing, get a **screen capture.** Add two blank spaces following the first screen capture and then paste this one.
6. **Copy** the **right side of the explicit form** (select it and select Edit/Copy or Ctrl+c).
7. Change to the Explicit page of the Sequence Editor and paste the explicit form following $\mathbf{a_n}$E. Remember to press EXE!
8. With the Explicit page and table showing, get a **screen capture.** Add two blank spaces following the second screen capture and then paste this one.

PART III

Sometimes it is useful to sum a sequence. The sum of the terms in a sequence is called a series. Series play an important role in Calculus; for now we will just look at a neat symbol used for summing sequences.

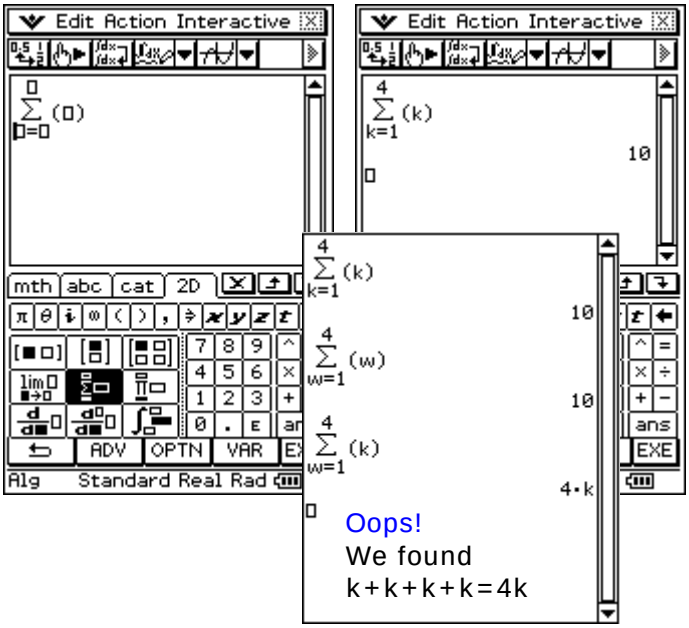
The summation symbol $\sum$ is the uppercase Greek letter sigma and is used as a short hand notation for showing the sum of a sequence. The ClassPad has one in 2-D CALC!



1. Using the ClassPad's Summation Symbol

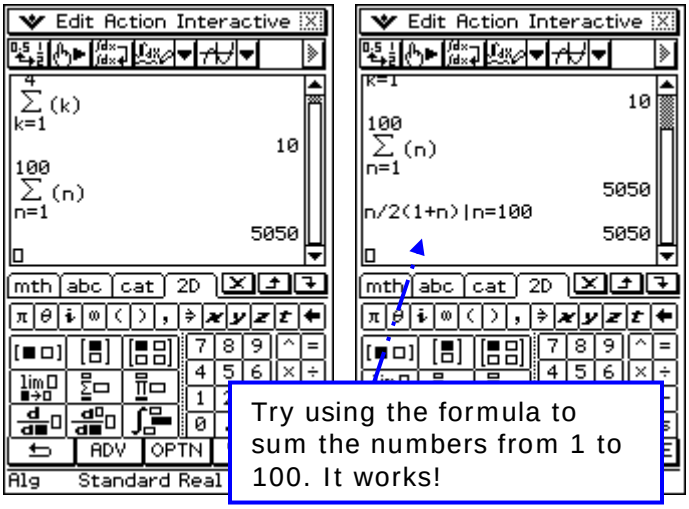
A few words of wisdom: Whenever you try something new on a calculator or software that gives a result, first try an example that you know the answer to!

a. Let's try summing the numbers from 1 to 4.
b. We know 1+2+3+4=10
c. Open Main ( ) and clear the window
d. Open the **keyboard** and find the summation symbol
e. We want to sum k for k=1 to 4 (see mine)
f. Find the sum!
g. The letter you use **should match** the letter in the sequence we are summing



2. Summing an Arithmetic Sequence

a. Find the sum of the numbers from 1 to 100.
b. Notice that this is a arithmetic sequence of the form $\mathbf{a_n = n}$ (each term differs from the next by the constant 1)
c. A shortcut for finding sums of arithmetic sequences from 1 to n is $S_n = \frac{n}{2}(a_1 + a_n)$.



PART III
Practice Exercises

1. Please open the Main application and clear the window.
2. Using summation notation, find the sum of the numbers in the sequence **a_n=3n-5** for **n=1 to 10**.
3. On the next line, find the sum using the formula for finding the sum of an arithmetic sequence. [**Hint**: Use a_1=3x1-5 and a_n=3x10-5.]
4. With both your sums showing, get a **screen capture.** Paste it into your Lesson18 document (under a title of PART III).
5. Open the eActivity named **L18_PartIII_a** in the **Lesson 18** folder.
6. Evaluate the summation in the Verify strip named Exercise 1. Be sure to use 4 lines to reach the final answer (see the example inside the eActivity for more details).
7. With your Verify window open, get a **screen capture.** Add two blank spaces following the first screen capture and then paste this one.
8. Save your eActivity as **L18_PartIII_a_*your initials.***
9. Open the eActivity named **L18_PartIII_b** in the **Lesson 18** folder.
10. Read through the eActivity and complete the summation inside the strip labeled **Summation**. This activity is design to show you that some summations converge to a single number. In Calculus, you will learn a great deal more about converging series.
11. With you Main window open and the summation showing **as a decimal**, get a **screen capture.** Add two blank spaces following the second screen capture and then paste this one.
12. Save your eActivity as **L18_PartIII_b_*your initials.***

PART IV

Reflection Exercises

You have just completed the eighteenth lesson in ClassPad 101. Excellent. You are almost done! Please take a few moments to copy and paste the following three questions at the end of your Lesson 18 document and answer them.

1. Approximately how long did it take you to complete this lesson?
2. Which activity did you find particularly useful?
3. Did you find any part of this activity difficult to follow? If so, which part? Also, how did you overcome the difficulty?

Assessment 18: Introduction to Sequence

- **Checkpoint**: Your word processed document, titled "Lesson18", should contain the following activities:
  1. Three screen captures from PART I.
  2. Three screen captures from PART II.
  3. Three screen captures from PART III.
  4. Three reflection questions with answers from PART IV.
- **Submit** your **Lesson18 document** to your instructor for grading.